***2.9 Time Advancement Mechanism***

***2.10 Queuing Models and Their Characteristics***

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***2.12 Measures of Queues (Single Server Queuing System)***

**Case Study: Queue Management in a Call Center**

In a busy call center, managing customer queries efficiently is a key challenge. The call center has a **single server** (one agent) and serves customers who call in throughout the day. This system can be modeled using a **single-server queuing model** (M/M/1 queue). The goal of this case study is to understand how time advancement mechanisms and queuing models can be used to optimize call handling, reduce customer waiting times, and improve service efficiency.

**The System:**

The call center has a steady stream of customers, arriving at random intervals. When a customer calls, the system places them in a queue if the agent is busy. The agent serves customers at a certain rate, and the time between serving each customer follows an exponential distribution.

Key characteristics of the system include:

1. **Arrival rate (λ)**: The rate at which calls arrive. For this case, assume an average of 10 customers call every hour.
2. **Service rate (μ)**: The rate at which the agent serves customers. Assume the agent serves an average of 12 customers per hour.

The **Time Advancement Mechanism** plays a crucial role in simulating the flow of customers in the queue. As customers arrive, the system advances time to the next event: either the arrival of a new customer or the completion of the service of the current customer. By advancing time through the system’s events, we can calculate and monitor the number of customers in the queue at each point and adjust operations accordingly.

**Queuing Characteristics:**

The system described is a typical **M/M/1 queuing model**, which assumes:

1. **Markovian arrivals** (Poisson process for customer arrivals).
2. **Markovian service times** (exponentially distributed service times).
3. A **single server**.

In such a system, we need to monitor several key metrics, such as the **utilization** of the server (how often the agent is busy), the **average number of customers in the queue** (to gauge the system's efficiency), and the **average time customers spend in the system** (to measure the waiting experience).

**Queuing Discipline:**

In this case, the **First-Come, First-Served (FCFS)** queuing discipline is used, meaning that customers are served in the order they arrive. This is the most common discipline in call centers, as it is perceived as fair and simple to implement. However, other disciplines such as **Priority Queueing** (where urgent calls are prioritized) could be considered depending on the nature of the calls (e.g., for customer complaints vs. routine inquiries).

**Measuring Queue Performance:**

Using the **M/M/1 model**, we can compute several performance measures:

1. **Utilization (ρ)**: The fraction of time the server is busy. This is calculated as **ρ=λ / μ**​
2. **Average number of customers in the system (L)**: The expected number of customers in the queue and being served. This can be calculated **as L=λ (μ−λ).**
3. **Average time a customer spends in the system (W)**: The expected time a customer spends from arrival to departure. This is calculated as **W=1 / (μ−λ)**
4. **Average number of customers in the queue (Lq**: The expected number of customers waiting in the queue. This can be calculated as **Lq = λ2 / μ (μ−λ)**
5. **Average time a customer spends waiting in the queue (Wq)**: The expected time spent waiting in the queue. This is calculated as **Wq=λ / μ(μ−λ).**

By understanding these metrics, the call center manager can make informed decisions on staffing levels, the need for additional agents, or other operational adjustments to improve service efficiency.

**Question:**

A call center has the following characteristics:

* The arrival rate of customers (λ) is 10 per hour.
* The service rate (μ) is 12 per hour.

1. What is the server utilization (ρ)?
2. Calculate the average number of customers in the system (L).
3. Calculate the average time a customer spends in the system (W).
4. Calculate the average number of customers in the queue (Lq​).
5. Calculate the average time a customer spends waiting in the queue (Wq​).

**Numerical Solution:**

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**Conclusion:**

By analyzing the queuing system using the **M/M/1 model**, the call center can see that the average number of customers in the system is 5, the average waiting time in the queue is approximately 25 minutes, and the server is busy 83.33% of the time. These insights help in managing resources more effectively and improving customer service.